BETWEEN INTERACTIONS AND INCENTIVES

Some works around Contract Theory

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OUTLINE

CONTRACT THEORY

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Moral Hazard: the Agent's behaviour is not observable by the Principal (Second–Best).

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Adverse Selection: a characteristic of the Agent is unknown by the Principal (Third–Best).

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- The optimal form of contracts for the Agent is (see Sannikov [2008]):

$$\xi = \xi_0 - \int_0^T \mathcal{H}(Z_s) ds + \int_0^T Z_s dX_s, \tag{1}$$

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where

(i) Z is a payment rate chosen by the Principal;

(ii) $\mathcal{H}$ is the Agent's Hamiltonian.

PRINCIPAL – MEAN-FIELD AGENTS

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- Electricity cannot be stored: supply–demand balance at all times.

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Infinity of consumers: Extension of Aïd et al. [2018] to a Mean–Field of consumers, whose consumption is impacted by a common noise representing the weather conditions.

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EXTENSION TO A MEAN–FIELD OF AGENTS

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▶ Restrict the study to a small consumer, similar to others, who has a negligible impact on the global consumption: representative Agent.

The output process is the deviation from its usual consumption:

$$X_t = x_0 - \int_0^t \alpha_s \cdot \mathbf{1}_d \mathrm{d}s + \int_0^t \sigma(\beta_s) \cdot \mathrm{d}W_s + \int_0^t \sigma^\circ \mathrm{d}W^\circ_s, \; t \in [0, T]. \tag{2}$$

where

- W, d–dimensional BM : Agent's own randomness;
- W°, 1–dimensional BM: common noise for all Agents;
- α, effort to reduce the mean of his consumption;
- β, effort to reduce the volatility.

A NEW FORM OF CONTRACT

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Intuition: In the MF case, the Principal can benefit from the additional information she has.

▶ She can compute the conditional law with respect to the common noise of others' deviation, denoted $\hat{\mu}$, and index the contract on it:

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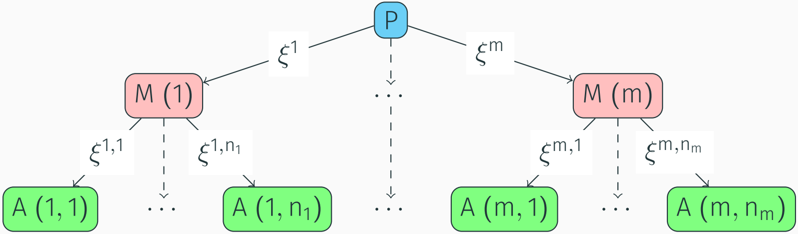
Main result: (i) equilibrium between Agents $\Leftrightarrow$ Mean–Field 2BSDE; (ii) this form of contracts, where the Principal choose $\zeta := (Z, \Gamma, Z^\mu)$, is optimal; (iii) Principal's problem $\Leftrightarrow$ McKean–Vlasov SDE.

OTHER APPLICATIONS

- The Principal (P) contracts with the Managers (M) who in turn, contracts with the Agents (A).



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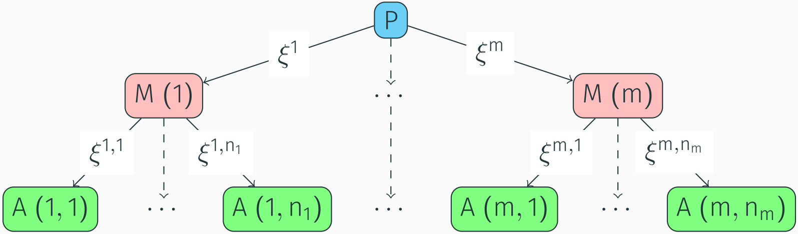

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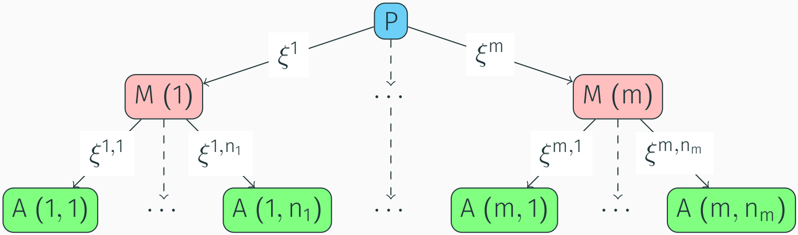

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Main result: even without volatility control by the Agents, it is not sufficient to limit the study to linear contracts since the Managers control the volatility of the "state variable" by choosing Agents' contracts.

- Use of 2BSDEs.

EPIDEMIC CONTROL: INDIVIDUAL POINT OF VIEW

- Élie, Hubert, and Turinici [2020].

$$\boxed{\text{Susceptible}} \xrightarrow{\beta_t S_t I_t dt} \boxed{\text{Infected}} \xrightarrow{\gamma I_t dt} \boxed{\text{Recovery}}$$

Figure: SIR model

- Dynamic of an epidemic SIR model:

$$\begin{cases} dS_t = -\beta_t S_t I_t dt, \\ dI_t = (\beta_t S_t I_t + \gamma I_t)dt, \\ dR_t = \gamma I_t dt, \end{cases} \quad \text{for } \; t \in [0, T].$$

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- Each individual can choose to decrease his social interactions with others (decrease β) $\Leftrightarrow$ Lockdown.

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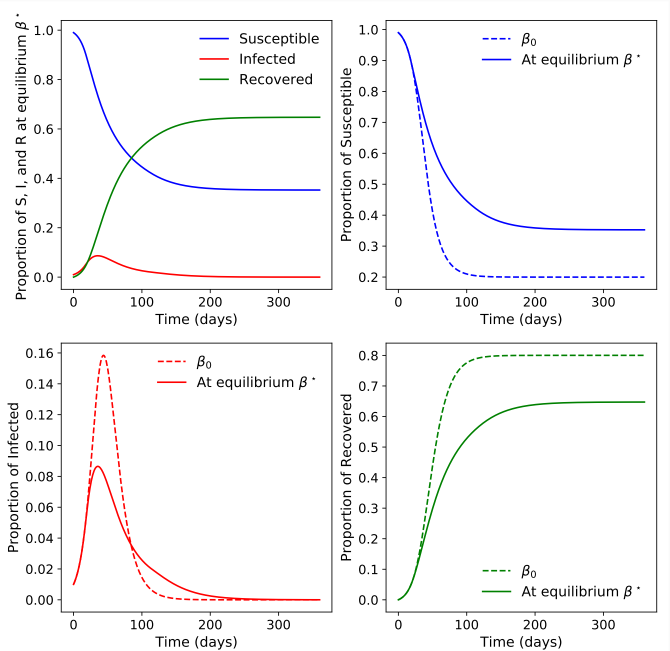
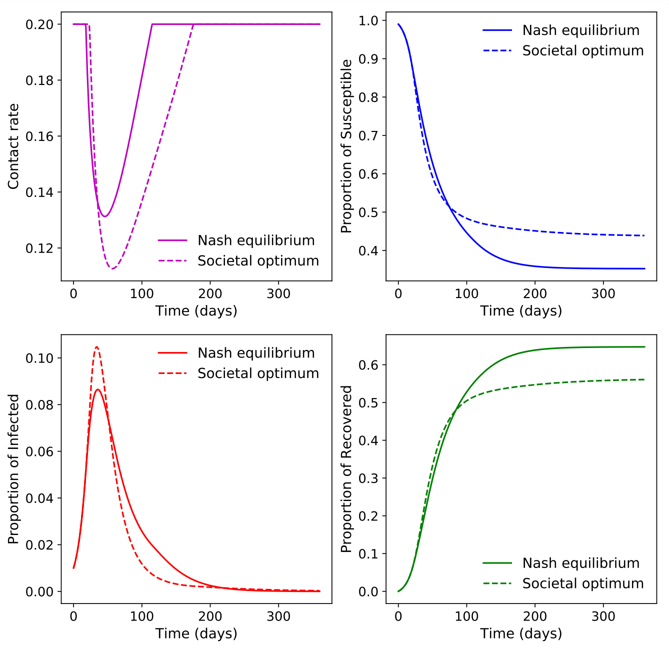
- Nash between individuals $\beta^\star$, different from societal optimum.

NASH EQUILIBRIUM



COMPARISON WITH SOCIETAL OPTIMUM



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EPIDEMIC CONTROL: GOVERNMENT'S POINT OF VIEW

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- Epidemic SIR model, stochastic version.

$$\begin{cases} dS_t = -\beta_t S_t I_t dt + \sigma S_t I_t dW_t, \\ dI_t = (\beta_t S_t + \gamma) I_t dt - \sigma S_t I_t dW_t, \quad \text{for } t \in [0, T]. \\ dR_t = \gamma I_t dt \end{cases}$$

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- The optimal form of contract satisfies

$$U(-\xi) = Y_0 - \int_0^T \mathcal{H}(S_t, I_t, Z_t) dt - \int_0^T Z_t dI_t.$$

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